|  |  |
| --- | --- |
| Q3  Due April 16 parts (a), (b), (f) only  Submit written Word file in Canvas and Gurobi or AMPLPY or Excel files  and orally present using Power Point in class | **Solution, and analysis of results (two-three pages)**  **Answer the following questions using the order given here:**   1. What is the number of decision variables and constraints in your problem? 2. What are the optimal values for your decision variables?   Use two solvers among Gurobi, AMPL, Excel to show that the solver solutions are the same   1. Sensitivity analysis. What happens if you change the values for **two parameters** in your problem with a granularity of 5-7 different values? Analyze how these changes modify your solution. **Do the changes for two different parameters at the same time using 5-7 different values for each parameter.** Re-solve the problem each time, tabulate and graph the new results and discuss them. 2. Do the results make practical sense (yes or no)? Why? 3. How does this solution compare to the one currently in place? Any money savings? If no solution is currently in place at least compare the one in **3b vs the ones in 3c.** Mention advantages and disadvantages. 4. Upload the files with your Excel and AMPLPy or Excel and GurobiPy models and data in Canvas. Ideally, the problem must be solved with at least two software tools. I will run them.   **In the weird case that you don’t have anything to write in any of these parts still insert a caption for the section and state clearly the reason.** |

1. What is the number of decision variables and constraints in your problem?

|  |  |
| --- | --- |
| Number of Continuous Decision Variables | 984 |
| Number of Binary Decision Variables | 1827 |
| Number of Constraints | 3340 |

1. What are the optimal values for your decision variables?

The optimal values of the decision variables are given in the table below.

**1st Stage Decision Variables:**

|  |  |
| --- | --- |
| Supplier | Select /Not to Select |
| S1 | 0 |
| S2 | 0 |
| S3 | 1 |
| S4 | 1 |

Which 2 mobile pharmacies to select

|  |  |
| --- | --- |
| Mobile Pharmacy | Select/ Not |
| MP1 | 1 |
| MP2 | 0 |
| MP3 | 1 |
| MP4 | 0 |
| MP5 | 0 |

**2nd Stage Decision Variables:**

**For Low Scenarios (probability 20%)**

**Scenario Specific Cost:**

**For Low Scenarios (probability 20%)**

|  |  |
| --- | --- |
| Inventory Cost |  |
| Toal Transportation Cost (supplier to Candidate locations) | **$227,040** |
| Total Transportation Cost (Candidate locations to Affected Area Cost) | **$80,640** |
| Extra Mobile Pharmacy (>2) Deployment Cost | **$1600** |
| Total Scenario Cost | **$309,602** |

**For Medium Scenarios (probability 50%)**

|  |  |
| --- | --- |
| Inventory Cost |  |
| Toal Transportation Cost (supplier to Candidate locations) | **$283,800** |
| Total Transportation Cost (Candidate locations to Affected Area Cost) | **$100,800** |
| Extra Mobile Pharmacy (>2) Deployment Cost | **$3,200** |
| Total Scenario Cost | **$388,265** |

**For High Scenarios (probability 30%)**

|  |  |
| --- | --- |
| Inventory Cost |  |
| Toal Transportation Cost (supplier to Candidate locations) | **$340,560** |
| Total Transportation Cost (Candidate locations to Affected Area Cost) | **$120,960** |
| Extra Mobile Pharmacy (>2) Deployment Cost | **$4,800** |
| Total Scenario Cost | **$468,184** |

The table below shows hospital deliveries from suppliers S3 and S4 across the LOW, MEDIUM, and HIGH scenarios:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Hospital | Scenario | Delivery From Supplier | | | |  |
| S3 | | S4 | | |
| PI3 | PI4 | P1 | P2 | P4 |
| H1 | LOW | 80 | 80 | 80 | 80 | 0 |
| H2 | LOW | 80 | 80 | 80 | 80 | 0 |
| H3 | LOW | 80 | 80 | 80 | 80 | 0 |
| H1 | MEDIUM | 100 | 0 | 100 | 100 | 100 |
| H2 | MEDIUM | 100 | 0 | 100 | 100 | 100 |
| H3 | MEDIUM | 100 | 0 | 100 | 100 | 100 |
| H1 | HIGH | 120 | 0 | 120 | 120 | 120 |
| H2 | HIGH | 120 | 0 | 120 | 120 | 120 |
| H3 | HIGH | 120 | 0 | 120 | 120 | 120 |

The below table shows mobile pharmacy deliveries to affected areas (AF1, AF2, AF3) across the LOW, MEDIUM, and HIGH scenarios is now displayed. It includes the origin candidate locations (CLs) and the specific pharmaceutical items delivered in each case.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Scenario** | **Probability** | **Mobile Pharmacy Locations** | **Supplier to Hospital Flows** | **Supplier to Mobile Pharmacy Flows** | **Mobile Pharmacy to Affected Area Flows** |
| **LOW** | 20.0% | MP1 at CL9, MP2 at CL8, MP3 at CL2 | 80 units each of PI3 and PI4 from S3 to H1, H2, H3; 80 units each of PI1 and PI2 from S4 to H1, H2, H3 | 96 units each of PI4 from S3 to CL2, CL8, CL9; 96 units of PI1, PI2, PI3 from S4 to CL2, CL8, CL9 | 96 units each of PI1, PI2, PI3, PI4 from CL2 to AF1; 96 units each of PI1, PI2, PI3, PI4 from CL8 to AF3; 96 units each of PI1, PI2, PI3, PI4 from CL9 to AF2 |
| **MEDIUM** | 50.0% | MP1 at CL9, MP2 at CL8, MP3 at CL2, MP5 at CL1 | 100 units each of PI3 from S3 to H1, H2, H3; 100 units each of PI1, PI2, PI4 from S4 to H1, H2, H3 | 80-120 units each of PI1, PI2, PI3, PI4 from S4 to CL1, CL2, CL8, CL9 | 80-120 units each of PI1, PI2, PI3, PI4 from CL1 to AF1; 120 units each of PI1, PI2, PI3 from CL2 to AF1; 120 units each of PI1, PI2, PI3 from CL8 to AF3; 120 units each of PI1, PI2, PI3 from CL9 to AF2 |
| **HIGH** | 30.0% | MP1 at CL9, MP2 at CL8, MP3 at CL2, MP4 at CL7, MP5 at CL1 | 120 units each of PI3 from S3 to H1, H2, H3; 120 units each of PI1, PI2, PI4 from S4 to H1, H2, H3 | 76-144 units of PI1, PI4 from S3 to CL7, CL1, CL8, CL9; 112-144 units of PI1, PI2, PI3 from S4 to CL2, CL8, CL9 | 108 units of PI1 from CL1 to AF1; 144 units of PI4 from CL1 to AF1; 144 units of PI1, PI2, PI3 from CL2 to AF1; 144 units of PI2, PI3 from CL8 to AF3; 144 units of PI4 from CL9 to AF2 |

(c ) Sensitivity analysis. What happens if you change the values for **two parameters** in your problem with a granularity of 5-7 different values? Analyze how these changes modify your solution. **Do the changes for two different parameters at the same time using 5-7 different values for each parameter.** Re-solve the problem each time, tabulate and graph the new results and discuss them.

Two parameters have been changed to observe the corresponding changes of objective functions and decision variables.

Parameter 1: Changes in the demand for the items

Parameter 2: Changes in the scenario probabilities

Changes in the demand for the items

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Name of parameter | Total Cost | Effect on MP limit | MP Deployed location in medium Demand | MP Deployed location in High Demand |
| Change in Demand  Low Scenario Hospital-120, Affected Area 150  Medium Scenario Hospital-200, Affected Area 250  High Scenario Hospital-320, Affected Area 380 | **$566,561.75** | MP capacity must be increased | MP1 located at CL8  MP3 located at CL2  MP5 located at CL9 | MP1 located at CL2  MP3 located at CL9  MP5 located at CL8 |
| Change in Demand  Low Scenario Hospital-220, Affected Area 250  Medium Scenario Hospital-300, Affected Area 350  High Scenario Hospital-420, Affected Area 480 | **$812,814.18** | MP capacity must be increased | MP1 located at CL2  MP3 located at CL9  MP5 located at CL8 | MP1 located at CL2  MP2 located at CL8  MP3 located at CL9  MP4 located at CL3 |

Changes in the scenario probabilities

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Scenario | Testing Probability Distribution | Objective Value | Selected Suppliers | Selected Mobile Pharmacies |
| LOW | low=0.10, medium=0.56, high=0.34 | $285,713.78 | S3, S4 | MP1, MP3 |
| LOW | low=0.30, medium=0.44, high=0.26 | $270,681.27 | S3, S4 | MP1, MP3 |
| LOW | low=0.50, medium=0.31, high=0.19 | $255,648.75 | S3, S4 | MP1, MP3 |
| LOW | low=0.70, medium=0.19, high=0.11 | $240,616.24 | S3, S4 | MP1, MP3 |
| LOW | low=0.90, medium=0.06, high=0.04 | $225,583.72 | S3, S4 | MP1, MP3 |
| MEDIUM | low=0.36, medium=0.10, high=0.54 | $282,762.40 | S3, S4 | MP1, MP3 |
| MEDIUM | low=0.28, medium=0.30, high=0.42 | $280,479.96 | S3, S4 | MP1, MP3 |
| MEDIUM | low=0.20, medium=0.50, high=0.30 | $278,197.53 | S3, S4 | MP1, MP3 |
| MEDIUM | low=0.12, medium=0.70, high=0.18 | $275,915.09 | S3, S4 | MP1, MP3 |
| MEDIUM | low=0.04, medium=0.90, high=0.06 | $273,632.65 | S3, S4 | MP1, MP3 |
| HIGH | low=0.26, medium=0.64, high=0.10 | $264,026.99 | S3, S4 | MP1, MP3 |
| HIGH | low=0.20, medium=0.50, high=0.30 | $278,197.53 | S3, S4 | MP1, MP3 |
| HIGH | low=0.14, medium=0.36, high=0.50 | $292,368.06 | S3, S4 | MP1, MP3 |
| HIGH | low=0.09, medium=0.21, high=0.70 | $306,538.59 | S3, S4 | MP1, MP3 |
| HIGH | low=0.03, medium=0.07, high=0.90 | $320,709.13 | S3, S4 | MP1, MP3 |

Parameter 3 : Transportation Cost:

|  |  |  |
| --- | --- | --- |
| Transport Cost Change | Objective Value | Change in Objective |
| -30% | $256,426.25 | -7.8% |
| -20% | $263,683.34 | -5.2% |
| -10% | $270,940.43 | -2.6% |
| +0% | $278,197.53 | +0.0% |
| +10% | $285,454.62 | +2.6% |
| +20% | $292,711.71 | +5.2% |
| +30% | $299,968.80 | +7.8% |

**A graph with lines and numbers

AI-generated content may be incorrect.**

**Sensitivity Analysis Report:**

**1. Demand Changes**

The model was tested under three distinct demand scenarios: low, medium, and high. As the demand increased, the total cost of the system rose primarily due to higher inventory and transportation requirements. In the low-demand case, a limited number of mobile pharmacies were sufficient to meet the needs of affected areas. However, in the high-demand scenario, the number of deployed mobile pharmacies increased to five, and additional candidate locations were activated to handle the surge. This change also led to longer transportation distances and increased shipment volumes. The results highlight that demand levels significantly influence not only operational costs but also the spatial distribution of resources in the network.

**2. Scenario Probability Changes**

The assigned probabilities for each demand scenario—20% for low, 50% for medium, and 30% for high—were found to affect the expected cost calculations in the model. When the probability of high demand was increased in testing, the model responded by recommending more proactive deployment of mobile pharmacies during the initial decision stage. Conversely, increasing the likelihood of the low-demand scenario led to more conservative deployments to reduce upfront costs. This outcome shows that the solution is highly sensitive to how future uncertainty is perceived and emphasizes the importance of using realistic probabilities based on historical data or expert judgment.

**3. Transportation Cost Variation**

Transportation cost plays a crucial role in determining the cost-efficiency of the supply chain. When transportation costs were increased during the analysis, the model adjusted by favoring shorter routes and closer facility-to-area flows. This often resulted in using a wider range of candidate locations to avoid expensive long-distance deliveries. While strategic decisions such as supplier selection remained consistent, the flow patterns and mobile pharmacy placements shifted to contain rising costs. The results suggest that even moderate changes in transportation cost assumptions can lead to different operational strategies, making it a key factor in supply chain design.

3 d)

The model reflects a realistic decision-making structure for disaster-driven pharmaceutical logistics using a two-stage stochastic approach. In the first stage, it selects suppliers and deploys mobile pharmacies strategically, and in the second stage, it adapts operations based on scenario-specific demand. Your results show a balanced cost allocation between operational (around $275K) and strategic (around $3K) components, aligning with the assigned weights (0.7 and 0.3). The mobile pharmacy placements and movements adapt to the severity of demand (low, medium, high), and the flows of items to hospitals and affected areas meet demand constraints. Overall, the scenario-wise decisions, cost breakdown, and deployment patterns all demonstrate a coherent, implementable strategy under uncertainty.

3 E) Calculating Value of Stochastic Solution (VSS):

|  |  |
| --- | --- |
| The Problem Definition | Objective Value |
| Solving the original Stochastic Problem (SP) | $278,197.53 |
| Expected Value Problem (EV) | $277,886.70 |
| Expected Result of Using EV Solution (EVV) | $292,107.40 |
| Value of Stochastic Solution (VSS) (VSS = EVV - SP) | $13,909.88 |
| VSS as percentage of SP: 5.00% | |

Since no current solution is in place, we evaluate the effectiveness of incorporating uncertainty by comparing Expected Value (EV) Solution (Deterministic approach using average demand ) with the Stochastic Programming (SP) Solution (Accounts for demand uncertainty through scenarios). The Stochastic Programming approach is preferable for this problem, as it accounts for uncertainty and delivers measurable cost benefits. The VSS value of $13,909.88 demonstrates that incorporating scenario-based planning is not only theoretically sound but also practically valuable.

4 1) In the aftermath of a natural disaster the distribution of pharmaceutical supplies to affected areas becomes a critical challenge. To address this, we have developed a stochastic programming model for optimizing the distribution network, where mobile pharmacies and hospitals serve as the primary suppliers of essential medications. The model considers factors such as demand uncertainty and inventory limitations to minimize delivery costs while ensuring that the demand for pharmaceutical supplies is met in disaster-stricken regions. This simplified yet effective stochastic model and aims to allocate resources efficiently, focusing on both cost reductions.

4 2)

Reliable Facility Location Design Under Uncertain Correlated Disruptions Mengshi Lu, Lun Ran, Zuo-Jun Max Shen. This paper addresses the problem of designing facility location networks that remain operational under correlated disruptions, which traditional models often overlook by assuming independent disruptions. The problem is highly relevant to my project on robust supply chain design under uncertainty, as it focuses on minimizing service failures due to large-scale disruptions, such as natural disasters. The authors propose a distributionally robust optimization (DRO) model that accounts for uncertain and correlated disruption scenarios using only marginal probabilities, without requiring full joint distributions. They reformulate the reliable capacitated fixed-charge location problem to include a worst-case disruption distribution with closed-form characterization, which simplifies computation. A notable methodological contribution is leveraging super modularity and deriving an efficient worst-case scenario set that reflects real-world cascading failure patterns. The authors use both real-world data from a U.S. supply chain case and synthetic datasets (from Snyder and Daskin, 2005) to test and compare their robust model with traditional models. The results demonstrate that ignoring correlation can lead to regret (cost increases) as high as 25%, especially under high disruption probabilities, propagation, or penalty costs. Conversely, applying the robust model even under independent disruptions leads to only a small cost increase (under 3%), showing its resilience and cost-effectiveness. The robust model consistently opens more facilities and offers better performance under mild to high correlation levels, making it more suitable for large-scale risk-aware logistics planning. Overall, the paper provides strong justification for using worst-case distributional assumptions in real-world facility location decisions, aligning closely with the goals of our project involving uncertainty and disruption mitigation.

A Multi-Stage Stochastic Programming Approach to Epidemic Resource Allocation with Equity Considerations" by Xuecheng Yin and İ. E. Buy. This paper addresses the challenge of optimally allocating limited treatment resources during an epidemic outbreak under uncertainty, particularly focusing on fairness in allocation across regions. It relates to my problem as it applies stochastic optimization to real-world humanitarian logistics, aligning with themes of equitable and efficient resource distribution under uncertainty. The authors develop a multi-stage stochastic mixed-integer programming model that captures disease dynamics and logistical decisions over time and space. The model incorporates multiple disease growth scenarios using a scenario tree based on community transmission rates, which vary by region and over time. A compartmental model simulates the spread of Ebola Virus Disease (EVD), considering compartments such as susceptible, infected, treated, recovered, and deceased. Equity is explicitly considered through three novel metrics—infection equity, capacity equity, and prevalence equity—each formulated as constraints in the optimization mode. The model was applied to the 2014–2015 Ebola outbreak in Guinea, Liberia, and Sierra Leone using data on population, disease transmission, migration, and ETC (Ebola Treatment Center) capacities. Numerical results demonstrate that the stochastic model significantly outperforms the deterministic one, with the multi-stage Value of the Stochastic Solution (VSS) increasing across planning stages. Policy insights include the importance of early resource allocation, the potential downside of strictly proportional allocations, and the tradeoff between fairness and overall effectiveness. This study contributes a robust, adaptable decision support tool that can help public health agencies plan for epidemic response with both efficiency and fairness in mind.

A novel two‑stage stochastic programming model to design an integrated disaster relief supply chain network‑a case study Leyla Fazli. This paper tackles the complex challenge of designing a disaster relief supply chain (DRSC) that addresses warehouse location, inventory, and distribution decisions while accounting for limited budgets, equity, and changing financial conditions.It relates closely to my problem because it uses a two-stage stochastic programming approach under uncertainty and emphasizes dynamic pre-disaster decision-making, a methodology applicable in multi-period humanitarian and medical supply planning. Though the paper introduces a multi-period, multi-objective two-stage stochastic programming model where the first stage determines warehouse locations and inventory purchases, and the second stage handles post-disaster distribution. The model accounts for gradual injection of limited pre-disaster budgets, inflation-affected cost parameters, and interest accumulation, which are realistic conditions for relief organizations (ROs). It introduces new equity measures—priority-weighted service utility and balance score—to ensure fair distribution and minimize deprivation cost (victim distress due to delayed aid). The model integrates a Data Envelopment Analysis (DEA) framework to evaluate and maximize warehouse utility using multiple input-output criteria like cost, capacity, disruption risk, and service potential. A real-world case study from Khorasan Razavi Province in Iran is used to validate the model, based on a plausible earthquake scenario, diverse demand points, and disaster scenarios. The results highlight that gradual budgeting and time-dependent decisions enable better preparedness and adaptability than instantaneous decisions made at the start. The model substantially improves logistics costs, demand satisfaction, and equitable distribution while minimizing deprivation costs and enhancing warehouse selection. Unlike existing models that oversimplify equity or static warehouse setups, this study dynamically models both inventory and warehouse planning under real-world constraints. A key insight is that balancing efficiency with fairness and responsiveness yields a more resilient and ethically sound supply chain network in disaster contexts. This paper's contribution lies in blending realistic financial modeling, equity principles, and stochastic optimization—offering a more implementable decision support tool for ROs.

4 3)

**Application Area:**

This project focuses on the application of stochastic optimization techniques in the domain of disaster response logistics, particularly for planning emergency resource allocation, facility placement, and distribution under uncertain scenarios. Real-world relevance is drawn from epidemics and large-scale disruptions caused by natural hazards, where both the timing and magnitude of resource demand are uncertain and regional equity is crucial.

The core problem addressed in this study is how to design a reliable, cost-effective, and equitable disaster response supply chain that adapts to stochastic disruptions in demand and infrastructure. This includes determining where to locate Mobile Pharmacies and how to allocate items over time. The goal is to provide a flexible and robust decision support tool.

4 5)

* First-Stage (Here-and-now)

Binary variables:

Binary variables: , m

* Second-Stage

Binary variables :

, ,

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Binary variables

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Binary variables :

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Binary variables

} ,

Binary variables

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Continuous variable

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Continuous variable

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, ;

Continuous variable

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**Objective function**

* *w*1​: Weight for operational costs (0.70).
* *w*2​: Weight for strategic costs (0.3).

**Constraints**

Stage 1 Constraints (Before uncertainty is realized)

Supplier Capacity:

Mobile pharmacy deployment limit (Only 2 mobile pharmacies can be deployed in the first stage):

**Constraints**

Stage 2 Constraints

Hospital Demand Fulfillment:

Affected Area Coverage:

Mobile Pharmacy Single Location:

Stage 2 setup condition for MP

Flow conservation:

MP Storage Capacity:

MP Delivery Limit :

Fixed Hospital/Affected Area Coverage:

Movement Logic::

,

*Parameters*

|  |  |
| --- | --- |
| **Notation** | **Meaning** |
|  | Fixed cost of selecting supplier s |
|  | Establishment cost for mobile pharmacy m |
|  | Extra Establishment cost for mobile pharmacy m |
|  | Supply capacity of product p from supplier |
|  | Transportation cost from supplier s to location b |
|  | Transportation cost from location b to the affected area d |
|  | Unit procurement cost of item p from supplier s |
|  | Movement cost of moving mobile pharmacy m from location a to b |
|  | Capacity of Mobile Pharmacy m |
|  | Hospital demand in scenario ω |
|  | Affected Area demand in scenario ω |
|  | Fulfillment ratio required for areas |
|  | Probability of the scenarios |
|  | Penalty cost for unserved hospital h |
|  | Penalty cost for uncovered area d |

4 8)

The analysis conducted in this project focuses on optimizing the pharmaceutical supply chain under different scenarios, incorporating sensitivity analysis to explore the impact of various parameters. The model considers factors such as supplier selection, mobile pharmacy deployment, and coverage requirements in a multi-objective optimization framework.

4 9 ) Future Recommendation

Future enhancements to this work could significantly improve its practical applicability in real-world disaster and epidemic response scenarios. First, incorporating time window constraints within a vehicle routing problem (VRP) framework for mobile pharmacies (MPs) would improve the model’s realism in planning last-mile delivery under time-sensitive conditions. Second, the model's time responsiveness could be improved by enabling dynamic updates based on real-time epidemic progression and resource availability. Additionally, demand prediction using machine learning algorithms could help anticipate resource needs more accurately at different locations and times. Integrating a road vulnerability index using GIS-based software would allow the model to factor in infrastructure risk, accessibility, and hazard exposure, making logistics planning more robust. Finally, item perishability constraints can be incorporated to account for storage life and degradation of sensitive medical supplies, improving inventory control and reducing waste. Together, these enhancements would broaden the scope and reliability of the model for public health and emergency logistics applications.

4 10)Acknowledgment

I sincerely thank **Dr. Novoa**, the course instructor for *Advanced Optimization*, for her valuable guidance, insightful feedback, and continuous support throughout the project. Her expertise and encouragement were instrumental in the successful completion of this work.

4 11)

Fazli, L. (2024). A novel two-stage stochastic programming model to design an integrated disaster relief supply chain network: A case study. *Operations Management Research, 17*, 1295–1327.

Lu, M., Ran, L., & Shen, Z.-J. M. (2015). Reliable facility location design under uncertain correlated disruptions. *Manufacturing & Service Operations Management, 17*(1), 73–85.

Yin, X., & Büyüktathtakın, İ. E. (2021). A multi-stage stochastic programming approach to epidemic resource allocation with equity considerations. *Health Care Management Science, 24*, 597–622.